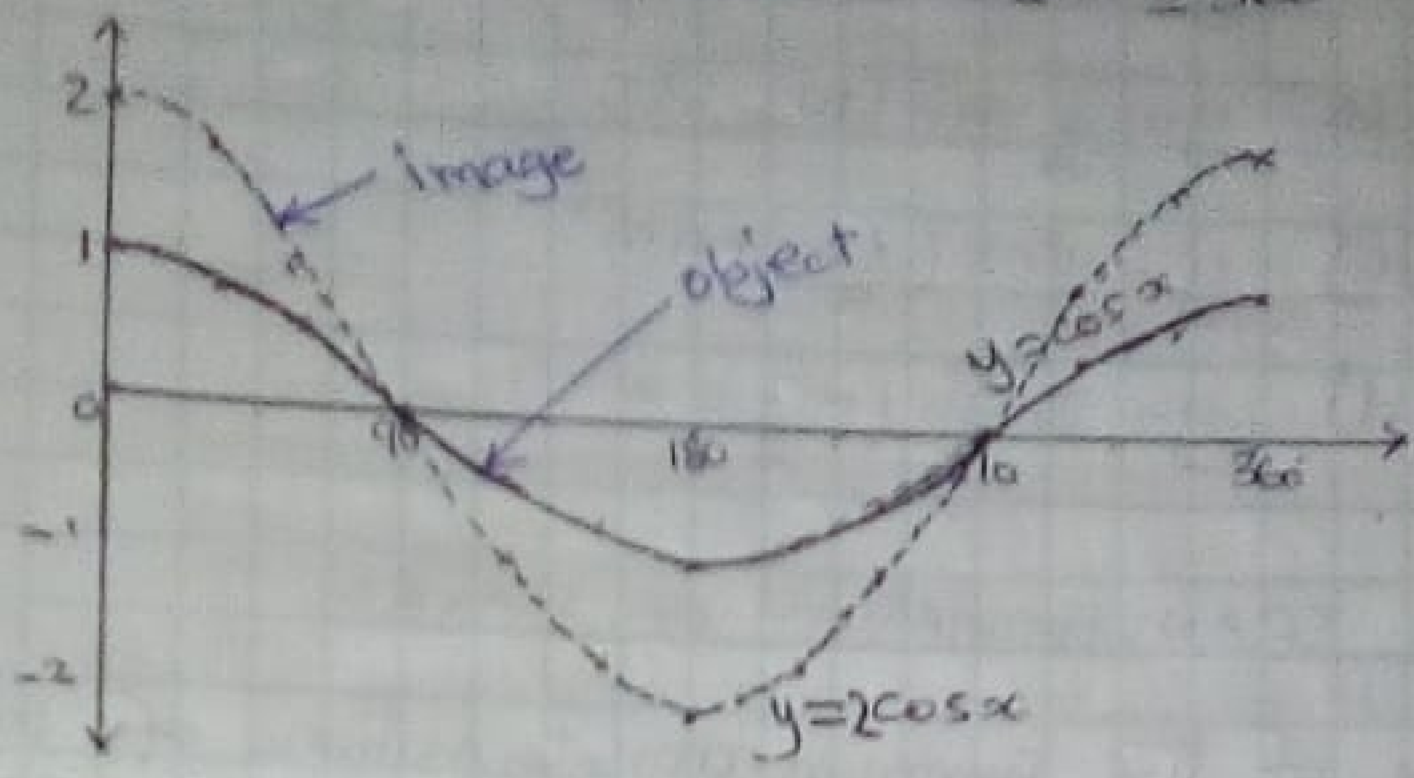
Example 2:

The figure below shows the graphs of $y = 2\cos x$ and $y = \cos x$ in the domain $0 \leq x \leq 360°$



The two waves have the same period (360°) but different amplitudes.

$y = \cos x$ can be transformed onto $y = 2\cos x$

by applying a stretch factor 2 with x-axis invariant.

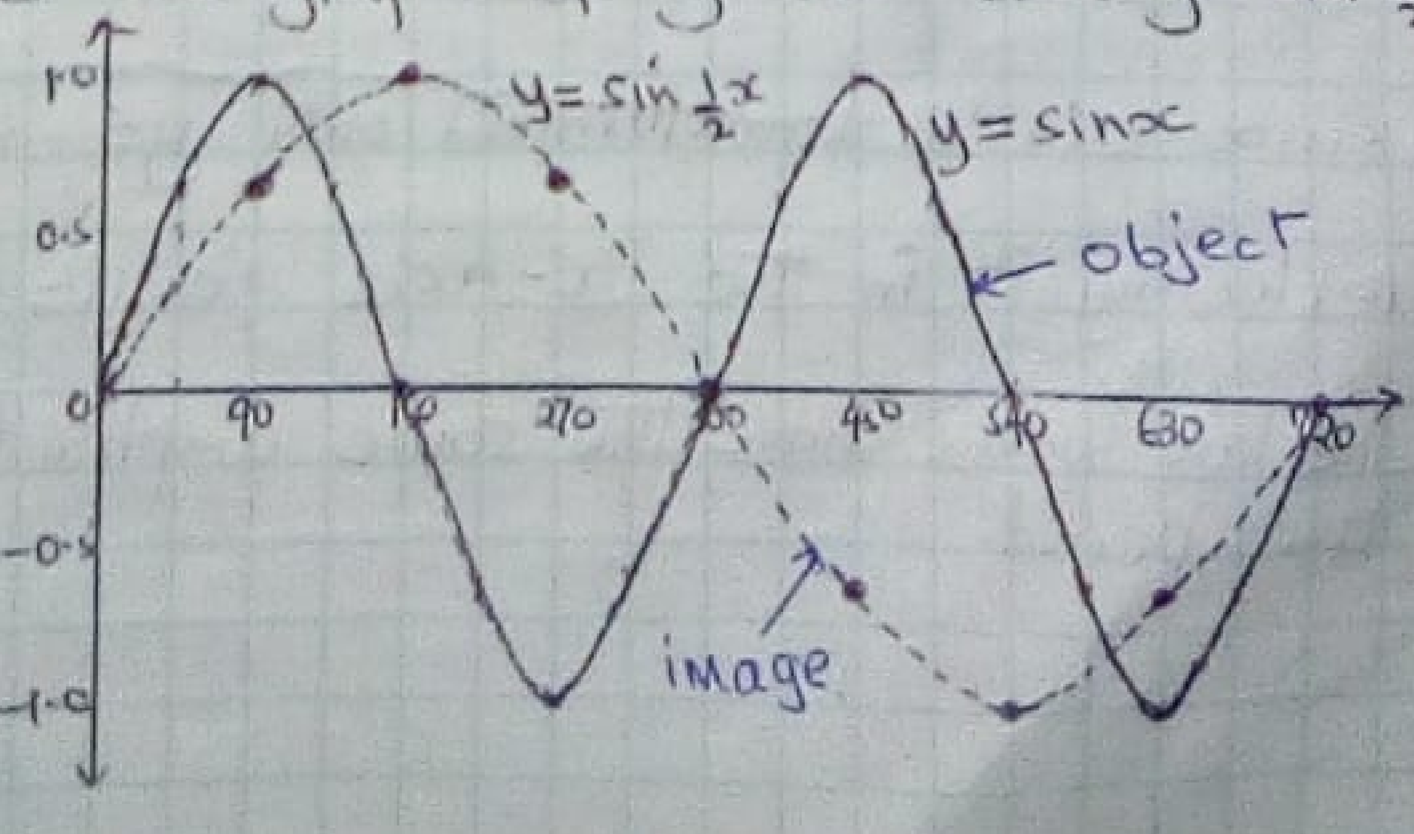
NB: stretch factor $= \dfrac{\text{amplitude of image}}{\text{amplitude of object}}$

$= \dfrac{2}{1}$

$= 2.$

Example 3:

Consider the graphs of $y = \sin x$ and $y = \sin \frac{1}{2}x$



The two waves have the same amplitude (1 unit) but different periods of 360° and 720° respectively.

$y = \sin x$ can be mapped onto $y = \sin \frac{1}{2}x$ by applying a stretch factor 2 with y-axis invariant.

NB. $\text{stretch factor} = \dfrac{\text{Period of image}}{\text{Period of object}}$

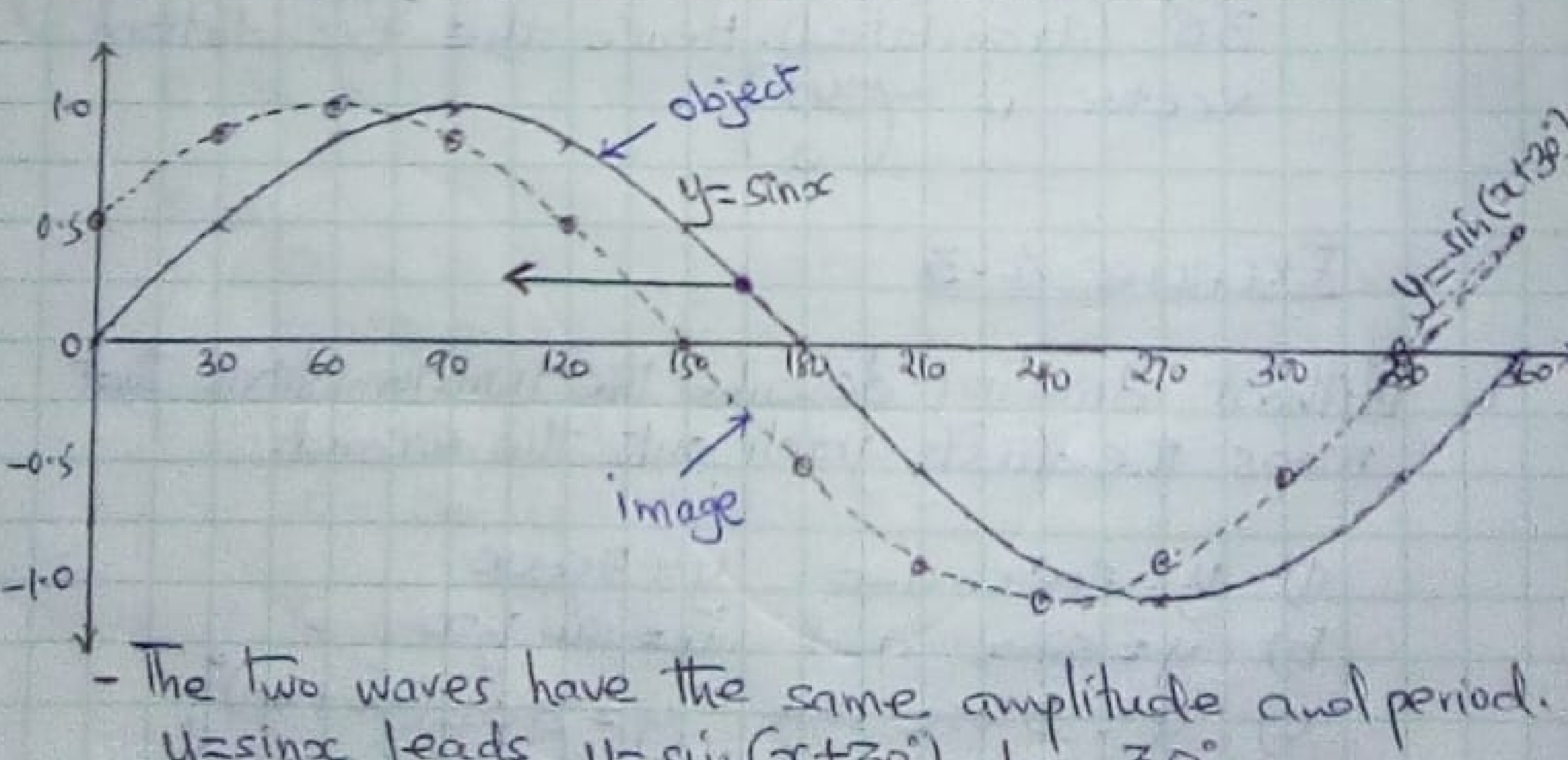
$= \dfrac{720°}{360°}$

$= 2.$

Example 4:

Consider the waves of $y = \sin x$ and $y = \sin(x + 30°)$.

$x°$	0	30	60	90	120	150	180	210	240	270	300	330	360	390
$y = \sin x$	0.0	0.50	0.87	1.00	0.87	0.50	0.0	-0.5	-0.87	-1.00	-0.87	-0.50	0.0	0.50
$y = \sin(x + 30)$	0.50	0.87	1.00	0.87	0.50	0.00	-0.50	-0.87	-1.00	-0.87	-0.50	0.00	0.50	0.87



- The two waves have the same amplitude and period. $y = \sin x$ leads $y = \sin(x + 30°)$ by 30°
- To map $y = \sin x$ onto $y = \sin(x + 30°)$ we translate (move) $y = \sin x$ to the left as shown by the arrow through 30°. ie (180° − 150°)
- Hence $y = \sin x$ can be mapped onto $y = \sin(x + 30°)$ by a translation of $\begin{pmatrix} -30 \\ 0 \end{pmatrix}$

NB:

(i) If both amplitudes and periods are different the transformation is a two-way stretch.

eg What transformation that would map $y = \sin x$ onto $y = 3 \sin 2x$

Soln.

	Amplitude	Period
$y = \sin x$	1	$360°$
$y = 3\sin 2x$	3	$180°$

Transformation is a stretch factor 3 with x-axis invariant followed by a stretch factor ½ with y-axis invariant.

(ii) In example 4 above, to map $y = \sin(x + 30°)$ onto $y = \sin x$, then $y = \sin(x + 30)°$ should be moved to the right through $30°$. (translated). Hence the translation vector is $\begin{pmatrix} 30 \\ 0 \end{pmatrix}$

EXERCISE 4.3

1. Without drawing describe the transformations that maps the first graph onto the second.

   a) $y = \cos x$ and $y = 5 \cos x$
   b) $y = \sin x$ and $y = \sin \frac{1}{4}x$
   c) $y = 3\cos x$ and $y = 3\cos(x + 45)°$
   d) $y = \sin x$ and $y = 4 \sin(x - 60)°$

2. Draw on the same axes the following graphs hence describe that maps the first graph onto the second.

   a) $y = \cos x$ and $y = \cos \frac{1}{3}x$ $0° \le x \le 720°$
   b) $y = \sin 2x$ and $y = 4 \sin 2x$ $0° \le x \le 360°$
   c) $y = \sin x$ and $y = \sin(x - 60)$ $0° \le x \le 540°$